

High efficiency Nondistortion Quantum Interrogation of atoms in quantum superpositions

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We consider the nondistortion quantum interrogation (NQI) of an atom prepared in a quantum superposition. By manipulating the polarization of the probe photon and making connections to interaction free measurements of opaque objects, we show that nondistortion interrogation of an atom in a quantum superposition can be done with efficiency approaching unity. However, if any component of the atom's superposition is completely transparent to the probe wave function, a nondistortion interrogation of the atom is impossible.

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Interaction-free measurements (IFM) were first considered by Elitzur and Vaidman to illustrate the peculiar nonlocality of quantum mechanics [1]. It was shown that it is possible to infer the presence of an absorbing object (in their original argument an ultra-sensitive “exploding bomb”) in a Mach-Zehnder interferometer without the probe photon being absorbed by the object. This works because the absorbing object blocks any photon passing it and changes the interference of the photon wave function. Since the original proposal of IFM, there have been many theoretical and experimental studies on this issue. It was shown that interaction-free measurements can in principle be done with unit efficiency in an asymptotic sense, for both opaque [2], [3] and semi-transparent objects [5], [6].

As emphasized by Vaidman [7], IFM's are not necessarily initial state preserving measurements. Due to the non-vanishing interaction Hamiltonian, in general IFM's can change very significantly the quantum state of the object being observed. However in most cases we wish to do the IFM without changing the internal state of the observed object, which we may call a “nondistortion quantum interrogation” (NQI) [8]. In most previous treatments, it was claimed that interaction-free measurements can be done for a quantum mechanical object as well as for the “exploding bomb” discussed in the original proposal [1]. For a two-level atom in its ground state interacting with a resonant photon, this is certainly true since the absorption of the photon destroys the initial state of the atom completely. However, the claim that IFM can be done equally well for a quantum mechanical object as for an “exploding bomb” is not fully justified unless the quantum superposition of the quantum object is taken into account. After all, the possibility of being in distinct states simultaneously is what distin-

guishes quantum from classical [9]. As discussed in a recent paper by Pötting *et. al* [10], the IFM and NQI of an atom in quantum superposition are more subtle than those of a classical object since the atom is subject to measurement dependent decoherence. Though in general NQI schemes can be designed for an atom in a quantum superposition [8], [10], the previous schemes based on a simple Mach-Zehnder interferometer setup yield very low success probabilities.

In this work we show that nondistortion interrogation of an atom in quantum superposition can be done with efficiency approaching unity, by using the model of [10] and making connections to IFM's of opaque objects. However, a necessary condition for such an NQI is that the possibility of interaction exists between the probe and every component of the superposition. It is then easily proved that an NQI of the atom in a quantum superposition is impossible if any component of the superposition is completely transparent to the probe.

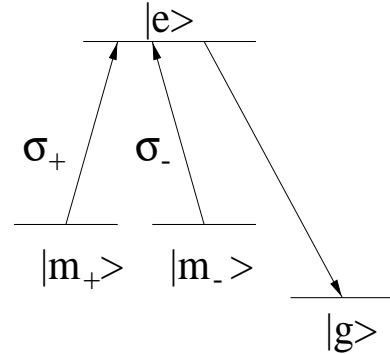


FIG. 1. Level structure of the atom. The atom can make a transition to the excited state $|e\rangle$ from $|m_+\rangle$ or $|m_-\rangle$ by absorbing a circularly polarized photon. It then decays rapidly and irreversibly to the stable ground state $|g\rangle$.

As in Fig. 1, the model we consider is a multilevel atom prepared in a superposition of the two degenerate metastable states $|m_+\rangle$ and $|m_-\rangle$. Starting from $|m_+\rangle$ and $|m_-\rangle$, the atom can absorb a $+$ or $-$ (circularly) polarized photon and make a transition to the excited state $|e\rangle$ with unit efficiency. It then decays irreversibly to the ground state $|g\rangle$ very rapidly. The whole process is

$$|\pm\rangle|m_{\pm}\rangle \longrightarrow |S_{\pm}\rangle|g\rangle \quad (1)$$

where $|\pm\rangle$ are the $+$ or $-$ polarized incident photons and $|S_{\pm}\rangle$ are the corresponding scattered photons which we assume will not be re-absorbed by the atom and can be filtered away from the detectors. The state of the atom is in the superposition

$$|\psi_{atom}\rangle = \alpha|m_+\rangle + \beta|m_-\rangle \quad (2)$$

where α and β are unknown non-vanishing amplitudes satisfying $|\alpha|^2 + |\beta|^2 = 1$.

If we can use a photon that will be completely absorbed by the atom, then the problem is identical to that of an opaque object. However, no matter what the photon's polarization is ($+$ or $-$ or a superposition of them), it will only be partially absorbed by the atom, due to the polarization selective interaction (1). For instance, the direct interaction between an x polarized photon $\frac{1}{\sqrt{2}}(|-\rangle - |+\rangle)$ and the atom results in the state

$$\frac{1}{\sqrt{2}}(\alpha|-\rangle|m_+\rangle - \beta|+\rangle|m_-\rangle) - \frac{1}{\sqrt{2}}(\alpha|S_+\rangle - \beta|S_-\rangle)|g\rangle \quad (3)$$

If the probe photon is not actually scattered, the photon and atom end up in the entangled state $\alpha|-\rangle|m_+\rangle - \beta|+\rangle|m_-\rangle$. As shown in [10], if we do not change the polarization of the photon through the interrogation process, this partial absorption and entangling will change the state of the atom even if no absorption happens, and result in a very low efficiency for the NQI of the atom.

At this point it might seem that an NQI of the atom in quantum superposition is similar to that of semi-transparent objects [6], since no complete absorption could happen if we do not do anything on the polarization of the photon. This is not true though. Once the wave functions of the photon and atom are entangled, the atom becomes transparent to the photon and it will not interact with the photon again when the photon passes it a second time. On the other hand, we can make a connection to NQI's of both opaque and semi-transparent objects if we let the photon pass the atom twice, with its polarization changed from the original value the second time. For instance, if we use a $+$ polarized photon to interact (directly) with the atom prepared in (2), we end up in the state $\beta|+\rangle|m_-\rangle + \alpha|S_+\rangle|g\rangle$ the first time. If no absorption actually happens, the photon and atom are left in $|+\rangle|m_-\rangle$. We then change the polarization of the photon to $-$ and let it pass the atom a second time. This time the photon will be absorbed by the atom with certainty. In this way the atom in superposition is effectively an opaque object to the photon. In the following, we show two ways of unit-efficiency (in an asymptotic sense) NQI of the atom in a superposition, following this idea of polarization rotation.

In Fig. 2 we consider the folded Mach-Zehnder interferometer discussed in [2]. For the purpose of clarity it is drawn in the form of N Mach-Zehnder interferometers connected in series, therefore the atom is in every

single interferometer (the dot). Each interferometer consists of two beam splitters ($BM1$ and $BM2$) and four reflecting mirrors ($R1$, $R2$ and $R3$, $R4$). $R3$ and $R4$ are used to redirect the photon to the atom after it passes the atom the first time. Suppose the probe is a $+$ polarized photon incident from the lower left port to the first interferometer. The reflectivity of each beam splitter is $R = \cos^2(\pi/2N)$ and the phase difference between the upper and lower paths is zero. In addition, the polarization of the photon is rotated to the orthogonal one (from $+$ to $-$ or from $-$ to $+$) when the photon travels between $R1$ and $R2$, $R3$ and $R4$ (There are many ways to do this, for instance by using a half wave plate). At $BM2$ the upper and lower branches of the photon wave function are in the same polarization (even though the polarization is orthogonal to that of the incident photon), so the interference between them is maintained. In absence of the atom, after N stages the photon will exit with certainty from the upper port of the last interferometer, with its polarization unchanged if N is even or rotated to the orthogonal value if N is odd.

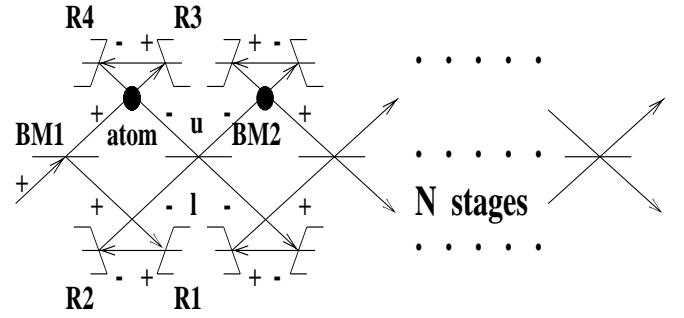


FIG. 2. Nondistortion interrogation of the atom with a series of Mach-Zehnder interferometers. Between $R1$ and $R2$, $R3$ and $R4$, the polarization of the photon is rotated to the orthogonal value. In absence of the atom, for properly chosen reflectivity of the beam splitters the photon will exit through the upper port of the last interferometer with certainty. When the atom is present, any photon going into the upper half of the interferometer is absorbed after it passes the atom twice. In this case there is a finite probability that the photon exits through the lower port of the last interferometer.

Now we see that the interference of the photon wave function is changed completely if the atom is in the interferometers (assume it is in the upper half of the system). Starting from the incident point, let us trace the wave function of the system (photon plus atom) until the photon arrives at $BM2$:

$$\begin{aligned} &|+\rangle_l(\alpha|m_+\rangle + \beta|m_-\rangle) \xrightarrow{BM1} (t|+\rangle_u + ir|+\rangle_l)(\alpha|m_+\rangle + \beta|m_-\rangle) \\ &\xrightarrow{atom} \alpha t|S_+\rangle|g\rangle + \beta t|+\rangle_u|m_-\rangle + ir|+\rangle_l(\alpha|m_+\rangle + \beta|m_-\rangle) \\ &\xrightarrow{R's} \alpha t|S_+\rangle|g\rangle - \beta t|-\rangle_u|m_-\rangle - ir|-\rangle_l(\alpha|m_+\rangle + \beta|m_-\rangle) \\ &\xrightarrow{atom} t(\alpha|S_+\rangle - \beta|S_-\rangle)|g\rangle - ir|-\rangle_l(\alpha|m_+\rangle + \beta|m_-\rangle) \quad (4) \end{aligned}$$

where l and u denote the lower and upper path and $(t, r) = (\sin(\pi/2N), \cos(\pi/2N))$ are the amplitude trans-

mission and reflection coefficients of the beam splitters. We have neglected the phase advance of the photon wave function in the above, since it is the same for the upper and lower branches (we assumed the photon wave function picks up a phase shift of i each time it is reflected). As expected, any photon that enters the upper half of the system is absorbed. On the other hand the reflected photon wave function is in a direct product with the initial state of the atom. So this is equivalent to the NQI of an opaque object, and after N stages the probability that the photon exits through the lower port (thus a successful NQI of the atom) is

$$P_{NQI} = [\cos^2(\pi/2N)]^N \quad (5)$$

which in the limit of large N goes to unity. As pointed out in [10], this way of unit-efficiency NQI can be viewed as a discrete form of the quantum Zeno effect [13].

Fabry-Perot interferometer can also be used to do NQI's of the atom [4], [6]. In Fig. 3, the incident photon is linearly (x) polarized. (The photon is assumed to be normally incident but for clarity it is depicted as if the angle of incidence was nonzero). In the Fabry-Perot interferometer, its polarization changes in the following way: when it goes through the upper half of the Fabry-Perot interferometer, its polarization is changed to $+$. The polarization is rotated to y when the photon goes through the lower half of the interferometer. When it is reflected back, its polarization is changed to $-$ and back to x . This can be done for instance by using a properly oriented half wave plate in the interferometer. So all the reflected and transmitted beams are in x and y polarization respectively. Assume the phase difference between adjacent reflected or transmitted beams is 4π (so all reflected and transmitted beams are in phase). Suppose the possible location of the atom is in the middle of the interferometer (represented by the dashed line). It is easily seen that when no atom is in the interferometer the interference of the reflected and transmitted beams is such that the photon goes through the interferometer with certainty, for any values of the amplitude reflection and transmission coefficients. In presence of the atom, in exactly the same way as described before the photon wave function that goes into the interferometer gets completely absorbed by the atom after it passes the atom twice. The final state of the photon-atom system is

$$ir|x\rangle_r(\alpha|m_+\rangle + \beta|m_-\rangle) + tt'\beta|y\rangle_t|m_-\rangle + |abs\rangle \quad (6)$$

where $|x\rangle_r$ and $|y\rangle_t$ are the reflected and transmitted photons (in x and y polarization) and $|abs\rangle$ (unnormalized) corresponds to the situation that the photon is absorbed. r is the amplitude reflection coefficient when the photon goes into the interferometer, t and t' are the amplitude transmission coefficients when the photon goes into and out of the interferometer. When the photon is reflected, the superposition of the atom is unperturbed and a successful NQI is realized. The probability of a successful NQI is $|r|^2$, which goes to unity when $|r| \rightarrow 1$.

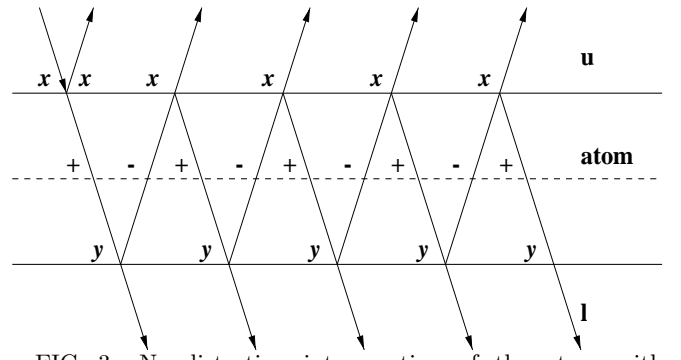


FIG. 3. Nondistortion interrogation of the atom with Fabry-Perot interferometer. Polarization of the photon is rotated from x to $+$, y , $-$ and back to x when the photon is reflected once in the interferometer.

In the above we showed that indeed high efficiency NQI's for atoms in quantum superpositions can be realized, through the connection to opaque objects. If we go beyond the model shown in Fig. 1 and consider other situations, for instance a system similar to that of Fig. 1 but with non-degenerate $|m_+\rangle$ and $|m_-\rangle$, what is the restriction in the more general cases? In the case of high efficiency IFM's for opaque objects, Kwiat *et. al* pointed out that in order to reduce the probability that an interaction occurs it is crucial that the *possibility* of such an interaction exists [3]. In the following we prove that the necessary condition for a successful NQI of an atom in a quantum superposition is that the possibility of interaction exists between the probe wave function and every component of the superposition.

We prove this by making use of a general formalism by Mitchson and Massar [6], with the additional requirement that the initial state of the atom must be kept unchanged. Suppose that the Hilbert space of the atom is an $N(\geq 2)$ dimensional space spanned by the orthonormal base vectors $\{|\Psi_{a,j}\rangle, j = 1, 2, \dots, N\}$. The NQI starts with $|\Psi_P^i\rangle|\Psi_a^i\rangle$, where $|\Psi_P^i\rangle$ and $|\Psi_a^i\rangle$ are initial states of the probe and atom respectively. The atom is prepared in the arbitrary and unknown superposition state

$$|\Psi_a^i\rangle = \sum_{j=1}^M a_j |\Psi_{a,j}\rangle \quad (7)$$

where a_j 's are unknown non-vanishing coefficients and $2 \leq M \leq N$. In the process of the interrogation, there are several steps in which the probe and atom are arranged in such a way that an interaction can potentially occur (the so called "I steps" in [6]). In between these steps unitary operations are performed on the probe wave function. The NQI fails and stops when an interaction between the probe and atom actually happens. If this is not the case the state of the probe is measured at the end. (A protocol in which the probe is measured before the end can be converted to this form [6], [11]). First consider the case that the atom is in the non-superposed state $|\Psi_{a,l}\rangle$ (all other components vanish). If the atom is not in the interferometer, no interaction between the photon and atom could

occur, and the effect of the NQI before the final measurement is an overall unitary operation on the probe: $|\Psi_P^i\rangle|\Psi_{a,l}\rangle \rightarrow |\Psi_{P,a,l}^f\rangle = U_P|\Psi_P^i\rangle|\Psi_{a,l}\rangle = |\Psi_P^f\rangle|\Psi_{a,l}\rangle$, where $|\Psi_P^f\rangle = U_P|\Psi_P^i\rangle$ and $l = 1, 2, \dots, N$. In presence of the atom, the interaction could happen, but the state of the atom will not be affected if the interaction does not actually happen (note this is only true for $|\Psi_{a,l}\rangle$'s but not for a superposition of them). So the final state is $|\Psi_{P,a,l}^{f'}\rangle = |\Psi_{P,l}^{f'}\rangle|\Psi_{a,l}\rangle + |\text{interacted}\rangle$ instead ($|\Psi_{P,l}^{f'}\rangle$ and $|\text{interacted}\rangle$ unnormalized), where $|\Psi_{P,l}^{f'}\rangle \neq |\Psi_P^f\rangle$ in general and $|\text{interacted}\rangle$ corresponds to the situation that an interaction happens. When the atom is in the superposition (7), the final state in absence of the atom is

$$|\Psi_{P,a}^f\rangle = |\Psi_P^f\rangle|\Psi_a^i\rangle = |\Psi_P^f\rangle \sum_{j=1}^M a_j |\Psi_{a,j}\rangle \quad (8)$$

On the other hand when the atom is present the final state is

$$|\Psi_{P,a}^{f'}\rangle = \sum_{j=1}^M a_j |\Psi_{P,a,j}^{f'}\rangle \quad (9)$$

We can see that the necessary condition that a successful NQI can be done is that there exists a projector $P = |\Phi_P\rangle\langle\Phi_P| \otimes I_a$ which satisfies $P|\Psi_{P,a}^f\rangle = 0$ and $P|\Psi_{P,a}^{f'}\rangle = \Delta|\Phi_P\rangle|\Psi_a^i\rangle$, where $|\Phi_P\rangle \neq 0$ is some state of the probe orthogonal to $|\Psi_P^f\rangle$, I_a is the unity operator for the atom and Δ is some nonzero number [12]. This is because an NQI requires that the probe can be measured in some final state orthogonal to $|\Psi_P^f\rangle$ (which reveals the atom's presence) with the atom's initial state unchanged. Now assume that some of the M components in (7), say $|\Psi_{a,i}\rangle$, $i = 1, 2, \dots, K$ ($K \leq M$) are completely transparent to the probe, either due to a vanishing interaction Hamiltonian between them or the design of the protocol. Then through the interrogation process the wave function of the system evolves as follows:

$$|\Psi_P^i\rangle \sum_{j=1}^M a_j |\Psi_{a,j}\rangle \rightarrow |\Psi_{P,a}^{f'}\rangle = |\Psi_P^f\rangle \sum_{j=1}^K a_j |\Psi_{a,j}\rangle + \sum_{j=K+1}^M a_j |\Psi_{P,a,j}^{f'}\rangle \quad (10)$$

Suppose the projector P for a successful NQI exists, the operation with P on $|\Psi_{P,a}^{f'}\rangle$ results in:

$$\begin{aligned} P|\Psi_{P,a}^{f'}\rangle &= \sum_{j=K+1}^M a_j P|\Psi_{P,a,j}^{f'}\rangle \\ &= \sum_{j=K+1}^M a_j \langle\Phi_P|\Psi_{P,j}^{f'}\rangle |\Phi_P\rangle |\Psi_{a,j}\rangle \end{aligned} \quad (11)$$

Obviously, an NQI in this case is impossible, since the right hand side of (11) does not contain any $|\Psi_{a,l}\rangle$, $l = 1, 2, \dots, K$ component. This is easy to understand, because the non-interaction between the probe and $|\Psi_{a,l}\rangle$ ($l = 1, 2, \dots, K$) makes it impossible to change the evolution of that branch of the wave function ($|\Psi_P^i\rangle|\Psi_{a,l}\rangle$). When the final state of the probe is measured using a projector orthogonal to $|\Psi_P^f\rangle\langle\Psi_P^f|$, all components that are completely transparent to the probe drop out of the atomic wave function.

This result explains why an NQI of the atom in superposition is impossible if a + or - photon is used as the probe and nothing is done on its polarization [10]. On the other hand, an NQI with linearly polarized probe photon is possible. (Actually our high-efficiency schemes work with linearly polarized photon too if its polarization is manipulated the same way we prescribed.) Also, an NQI for a system similar to that in Fig. 1 but with nondegenerate $|m_+\rangle$ and $|m_-\rangle$ is impossible if one uses a single probe photon in resonance with one (but not both) of the two metastable states $|m_+\rangle$, $|m_-\rangle$.

In summary, we showed that a nondistortion interrogation of an atom in a quantum superposition can be done with efficiency approaching unity, by making the photon wave function interact with all components of the superposition and turning the problem to that of an opaque object. On the other hand if any component of the superposition is transparent to the probe wave function, such an NQI is impossible.

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